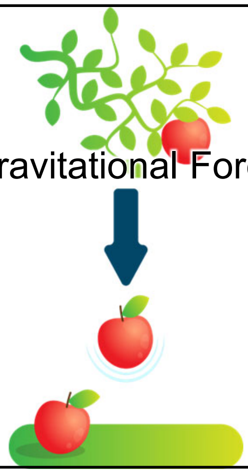
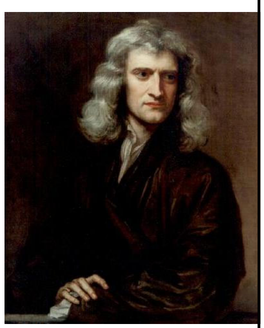
Gravitational Force



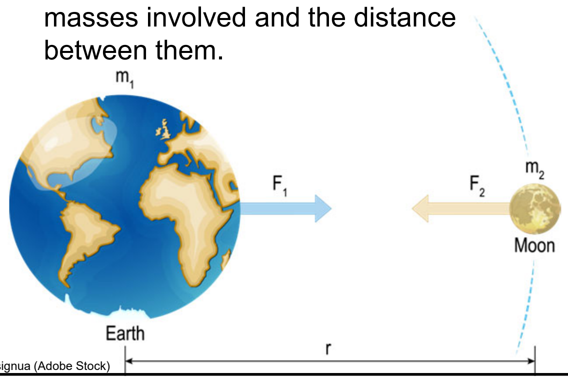
VectorMine (Adobe Stock)

- Sir Isaac Newton (British) was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions.



Godfrey Kneller (public domain)

- The gravitational force is always attractive, and it depends only on the masses involved and the distance between them.



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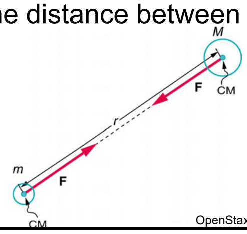
Newton’s Law of Universal Gravitation

- Every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.



- The magnitude of the force on each object (one has larger mass than the other) is the same, consistent with Newton's third law.
- The bodies we are dealing with tend to be large.
- To simplify the situation we assume that the body acts as if its entire mass is concentrated at one specific point called the center of mass (CM).

- For two bodies having masses M and m with a distance r between their centers of mass, the gravitational force between them is

$$\boxed{\vec{F}_g = G\frac{Mm}{r^2}}$$

G is the universal gravitational constant.

$$\boxed{G = 6.67 \times 10^{-11}\ \mathrm{Nm}^2/\mathrm{kg}^2}$$

- The universal gravitational constant G is determined experimentally.
- This determination was first done accurately in 1798 by English scientist Henry Cavendish (British).

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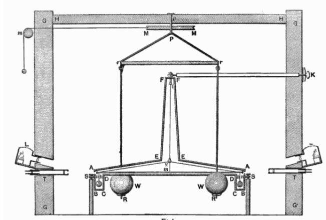
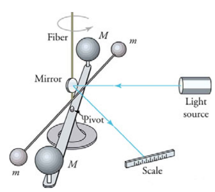
- Cavendish's experiment measured the tiny gravitational attraction between two ordinary-sized masses (tens of kilograms at most).



Henry Cavendish, Public Domain



OpenStax, Rice University (CC BY4.0)

Example 1

- What is the magnitude of the force acting on a 2000 kg spacecraft when it orbits the Earth at a distance of twice the Earth's radius?
  - Radius of Earth = 6380 km
  - Mass of Earth = 5.98×10^{24} kg

$$\vec{F}_g = G\frac{Mm}{r^2}$$

$$\vec{F}_g = (6.67 \times 10^{-11})\frac{(5.98 \times 10^{24})(2000)}{(2 \cdot 6380 \times 10^3)^2} = 4900\ \mathrm{N}$$

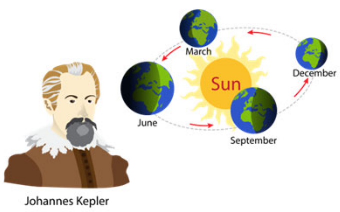
Example 2

- Two 70 kg people are sitting on a bench so that their centers of mass are 50 cm apart. What is the magnitude of the gravitational force each exerts on the other?

$$\vec{F}_g = G\frac{Mm}{r^2}$$

$$\vec{F}_g = (6.67 \times 10^{-11})\frac{(70)(70)}{(0.5)^2} = 1.3 \times 10^{-6}\ \mathrm{N}$$

- Newton's Law of Universal Gravitation provided a theoretical basis for Kepler's Third Law of planetary motion.



Kepler: Nattapon (Adobe Stock Photo)
Newton: Blueringmedia (Adobe Stock Photo)

Deriving Kepler's Third Law

- Consider a small mass m in a circular orbit about a large mass M.

$$F_{net} = \frac{mv^2}{r} \qquad v = \frac{2\pi r}{T}$$

$$F_{net} = \frac{m4\pi^2 r}{T^2}$$

- The net external force is caused by gravity.

$$G\frac{Mm}{r^2} = \frac{m4\pi^2 r}{T^2}$$

- Solving for T^2 gives

$$T^2 = \frac{4\pi^2}{GM} r^3$$

- Dividing by r^3 gives

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

Gravitational Field Strength

- We can use Newton's Law of Universal Gravitation to calculate the gravitational field strength (also called gravitational acceleration), g.
- The weight of an object of mass M is defined as the gravitational force on the object.

$$F = Mg = G\frac{Mm_{earth}}{r^2}$$

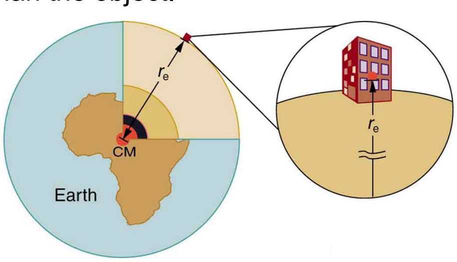
$$\boxed{g = G\frac{m_{earth}}{r^2}}$$

Substituting in the mass and radius of earth gives

$$g = (6.67 \times 10^{-11})\frac{(5.98 \times 10^{24})}{(6.38 \times 10^{6})^2}$$

$$g = 9.80\ m/s^2$$

- The distance between the centers of mass of Earth and an object on its surface is very nearly the same as the radius of Earth, because Earth is so much larger than the object.



Example

- Mt. Everest is 8848 m above sea level. Determine the gravitational field strength at the top of the mountain.
  - Radius of Earth = 6.30×10^6 m
  - Mass of Earth = 5.98×10^{24} kg

$$g = G\frac{m_{earth}}{r^2}$$

$$g = (6.67 \times 10^{-11})\frac{5.98 \times 10^{24}}{(6.38 \times 10^6 + 8848)^2}$$

$$g = 9.77\ m/s^2$$

Inertial Mass & Gravitational Mass

- **Inertial mass** is defined by Newton's Second Law, where a known force is applied to the mass and the acceleration it induces is measured.

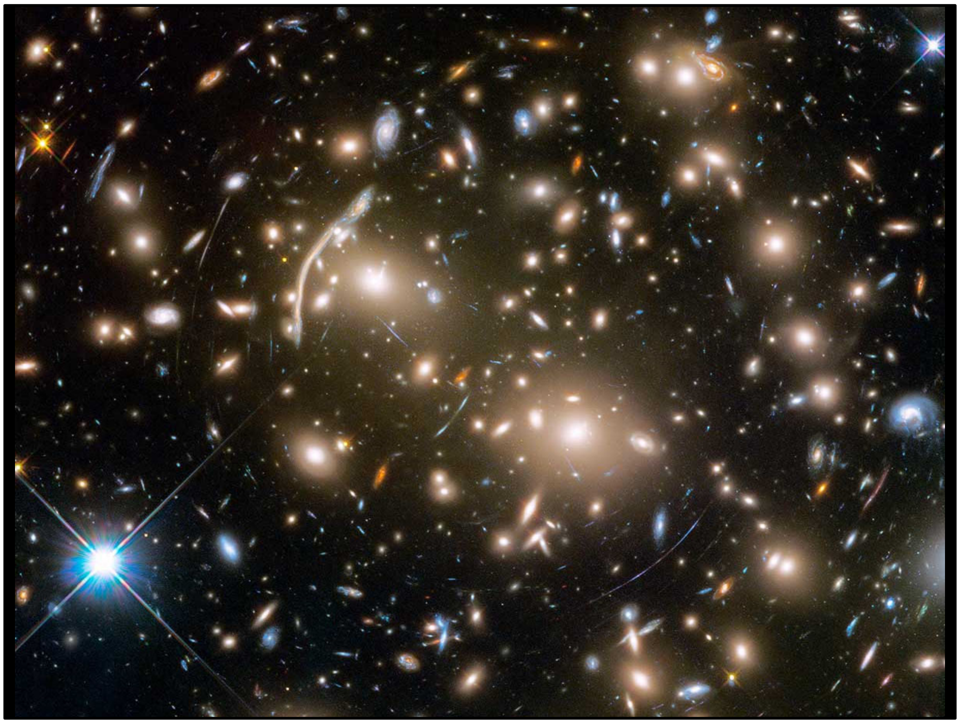
$$m = \frac{F}{a}$$

- **Gravitational mass** is defined by Newton's Law of Universal Gravitation.

$$F = G\frac{Mm}{r^2}$$

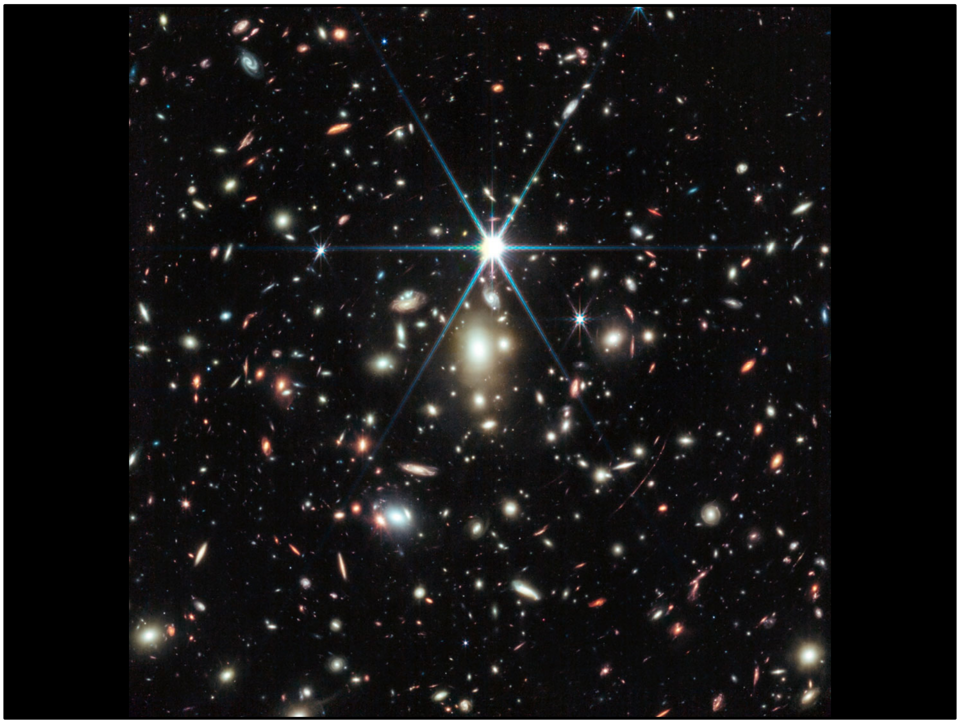
- It is measured by comparing the force of gravity of an unknown mass to the force of gravity of a known mass. This is typically done with some sort of balance scale.

- No difference has been found between gravitational and inertial mass.
- Many experiments have been performed to check the values and the experiments always agree to within the margin of error for the experiment.
- Einstein used the fact that gravitational and inertial mass were equal to begin his Theory of General Relativity in which he postulated that gravitational mass was the same as inertial mass.



Galaxy cluster Abell 370, located about 4 billion light-years away, contains an astounding assortment of several hundred galaxies tied together by the mutual pull of gravity. Entangled among the galaxies are mysterious-looking arcs of blue light. These are actually distorted images of remote galaxies behind the cluster. These far-flung galaxies are too faint for Hubble to see directly. Instead, the gravity from the cluster acts as a huge lens in space that magnifies and stretches images of background galaxies like a funhouse mirror. Nearly 100 distant galaxies have multiple images caused by the lensing effect. The most stunning example is "the Dragon," an extended feature that is probably several duplicated images of a single background spiral galaxy stretched along an arc. Astronomers chose Abell 370 as a target for Hubble because its gravitational lensing effects can be used for probing remote galaxies that inhabited the early universe.
Credit: NASA, ESA, and J. Lotz and the HFF Team (STScI)

The gravity of a luminous red galaxy (LRG) has gravitationally distorted the light from a much more distant blue galaxy. More typically, such light bending results in two discernible images of the distant galaxy, but here the lens alignment is so precise that the background galaxy is distorted into a horseshoe – a nearly complete ring.
Credit: ESA/Hubble & NASA



This image from NASA's James Webb Space Telescope of a massive galaxy cluster called WHL0137-08 contains the most strongly magnified galaxy known in the universe's first billion years: the Sunrise Arc, and within that galaxy, the most distant star ever detected.

The star, nicknamed Earendel, was first discovered by the Hubble Space Telescope. Follow-up observations using Webb's NIRCam (Near-Infrared Camera) reveals the star to be a massive B-type star more than twice as hot as our Sun, and about a million times more luminous.

Earendel is positioned along a wrinkle in spacetime that gives it extreme magnification, allowing it to emerge into view from its host galaxy, which appears as a red smear across the sky. The star is detectable only due to the combined power of human technology and nature via an effect called gravitational lensing.

In this image, the Sunrise Arc appears just below the diffraction spike at the 5 o'clock position. The fuzzier, white galaxies at the center of the image are part of the galaxy cluster bound together by gravity. The various redder, curved galaxies are background galaxies picked up by Webb's sensitive mirror.

Credits: Image: NASA, ESA, CSA, D. Coe (STScI/AURA for ESA; Johns Hopkins University), B. Welch (NASA's Goddard Space Flight Center; University of Maryland, College Park). Image processing: Z. Levay.